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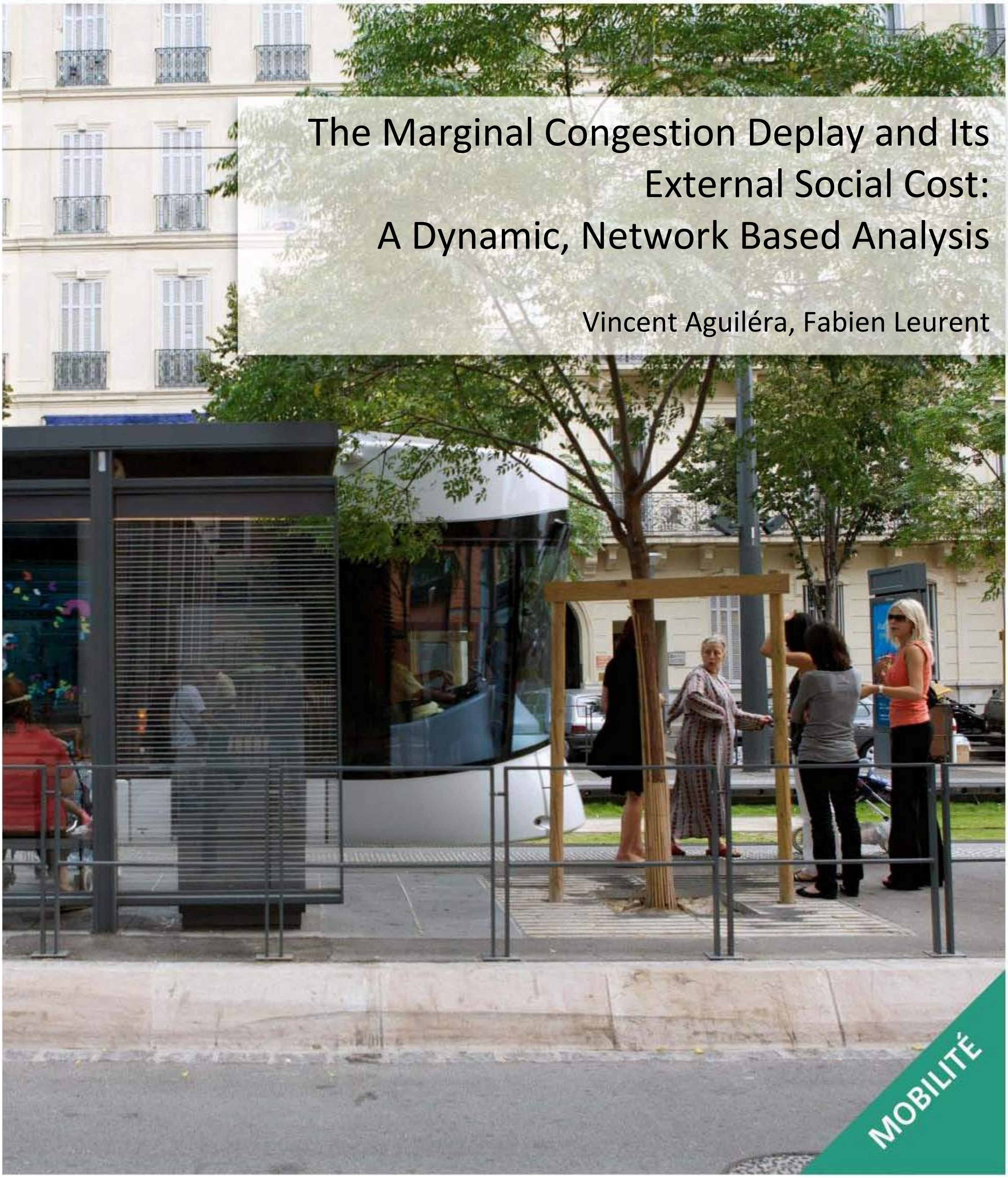

The Marginal Congestion Deplay and Its External Social Cost: A Dynamic, Network Based Analysis

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THE MARGINAL CONGESTION DELAY AND ITS EXTERNAL SOCIAL COST: A DYNAMIC, NETWORK BASED ANALYSIS

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ABSTRACT

On a congested transport network, a marginal trip exerts a congestion effect on the rest of traffic. From the polluter-pays principle in economic theory, the associated cost must be charged to the marginal trip-maker to achieve system optimum. This makes the evaluation of the marginal congestion cost a valuable objective, though not an easy one due to the dynamic and spatial nature of traffic phenomena. Besides the classical, static treatment, some dynamic models have been developed, most of them restricted to a fixed-capacity bottleneck. This paper develops the dynamic analysis of the social cost of congestion in two directions. First, analytical formulae are provided to deal with a multiclass flow in a bottleneck with time-varying capacity. Then, a set of increasingly complex situations are investigated: the sequence of bottlenecks along a route in the network; the set of routes of a given origin-destination pair. Lastly, helpful dynamic congestion indicators are designed for sub-networks and trip-end zones. An application to the major motorway network in France is given for the purpose of illustration.

THE MARGINAL CONGESTION DELAY AND ITS EXTERNAL SOCIAL COST: A DYNAMIC, NETWORK BASED ANALYSIS

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INTRODUCTION

The extension of congestion over transportation networks is a common feature of many cities and interurban corridors. Several schemes have been proposed and implemented for congestion management. Among them one can find: capacity increase, dynamic user information, high occupancy lanes, junction regulation, ramp-metering, congestion pricing. Some of those schemes follow the economists' recommendation to minimize the global social cost of traffic and congestion, including travel expense, time loss, safety, ecologic impacts. Following Vickrey *(1)*, social efficiency is reached when a trip is charged by a fee equal to the marginal social cost that this trip imposes to the other stake-holders. On a single arc of a transportation network, the question has been addressed by the economists' community. Several papers clearly state how the marginal social cost is to be computed, including the case where the dynamics of congestion is taken into account (see for instance *(2), (3)* and *(4)*). The situation is less clear for a whole network, but is of greater interest. It is less clear because trips emanating from various origin-destination pairs interact in intricate ways, both in space and time. It is of greater interest since, because of those complex interactions, a management scheme implemented on a single arc may impact large parts of a congested network, both in space and in time.

With dynamic congestion pricing in mind, the theory of marginal cost pricing has been used to design system-optimum time varying tolls on congested networks (see for instance *(5)* or *(6)*) with departure time choice. This paper investigates the design of indicators to assess the social cost of congestion induced by a marginal trip in a transportation network under user optimum dynamic equilibrium. In the authors' opinion, indicators like those provided in this paper can help transportation planning analysts in order to estimate the efficiency, location and scope of dynamic congestion management schemes. The emphasis is put on *ex ante* evaluation, on the basis of outputs from dynamic traffic assignments. Starting from the foundations, i.e. well established material for the static analysis of social marginal cost of congestion (SMC) on a single arc, the paper progressively investigates increasingly complex dynamic cases: single arc, single path, single o-d pair, sub-network, and trip-end zone. An application to the major motorway network in France is used for illustration purposes.

This paper comprises four sections. Section 1 briefly recalls the principles underlying the static analysis of the SMC. Section 2 exposes the method we propose for the multi-class, dynamic analysis of the SMC on a single arc. An example is provided to illustrate the noticeable difference one can find when comparing both methods. Section 3 extends the results of section 2 to the case of a transportation network, and provides methodological guidance for the identification of critical o-d pairs. Section 4 deals with the design of indicators by o-d pair and departure time.

1. STATIC ANALYSIS

A widespread method to evaluate the SMC is based on a static model and volume-delay functions *(7)*. First, this approach is sketched for a single class of users (subsection 1.1). Then, extensions to multiple user classes and several time periods are provided (subsections 1.2 to 1.4). Finally, the main limits of the static analysis are recalled (subsection 1.5).

1.1 The static model

The travel time t_a through a an arc a is modelled as an increasing function of the arc flow x_a. Well-known instances are the BPR volume delay *(8)* family of functions:

$$t_a(x_a) = t_{0,a}\left(1+\gamma_a\left(\frac{x_a}{\kappa_a}\right)^{\beta_a}\right) \quad (1)$$

where $t_{0,a}$ denotes the free-flow travel time that would prevail in the absence of traffic hence congestion, κ_a denotes the capacity flow, and β and γ are shape parameters (typically $\beta = 4$ and $\gamma = 0.4$). Other functions can be considered. See for instance *(9)*.

Let α denote the mean time-to-price trade-off ("value of time") of the users along arc a. Then the total user cost along a is:

$$C_a(x_a) = \alpha\, x_a t_a(x_a) \quad (2)$$

The total differential of C_a is

$$\frac{dC_a}{dx_a} = \alpha t_a + \alpha x_a \frac{dt_a}{dx_a}. \quad (3)$$

αt_a is the cost supported by the marginal user. The social marginal cost inflicted to other users amounts to $\alpha x_a dt_a / dx_a$. The social marginal cost inflicted to another user is $\alpha dt_a / dx_a$.

1.2 The case of several user classes

Let U a set of user classes. $\mathbf{x}_a = (x_{a,u})_{u\in U}$ denotes the vector of arc flows. A class u user experiences a travel time $t_{au}(\mathbf{x}_a)$. The class users experience a total congestion cost of $C_{a,u} = \alpha_u x_{a,u} t_{a,u}(\mathbf{x}_a)$, with α_u the time-to-price trade-off of class *u*. A marginal user of class *v* induces on class *u* a marginal total cost of

$$\frac{\partial C_{a,u}}{\partial x_{a,v}} = \delta_{u,v}\alpha_v t_{a,v} + \alpha_u x_{a,u}\frac{\partial t_{a,u}}{\partial x_{a,v}} \quad (4)$$

where $\delta_{u,v} = 1$ if $u = v$ or 0 otherwise. Thus the social marginal cost induced by a marginal user of class *v* user to all users amounts to $\sum_{u\in U} \alpha_u x_{a,u} \partial t_{a,u} / \partial x_{a,v}$.

1.3 The overall social marginal cost of congestion

As there are $x_{a,v}$ users of class *v* in the flow along arc *a*, on the whole they induce on the target class *u* a global social marginal cost of

$$\mathrm{SMC}(v \to u) = x_{a,v} \alpha_u x_{a,u} \frac{\partial t_{a,u}}{\partial x_{a,v}}$$

The overall SMC amounts to

$$(\alpha \circ \mathbf{x}_a).\nabla \mathbf{t}_a.\mathbf{x}_a = \sum_v x_{a,v} \sum_u \alpha_u x_{a,u} \frac{\partial t_{a,u}}{\partial x_{a,v}} \quad (5)$$

Where $\circ$ denotes the component-wise product. The overall SMC can be interpreted as the revenue from a congestion toll such that every user of class *v* would be charged a fee of

$$\pi_{a,v} = \sum_u \alpha_u x_{a,u} \frac{\partial t_{a,u}}{\partial x_{a,v}} \quad (6)$$

in order to cover the cost inflicted to other users.

1.4 The case of several periods

Let *P* denotes a set of time periods (e.g. the morning peak, the midday and the evening peak). Each period *p* in *P* is characterised by different travel times and congestion costs. Then the multi-period, multi-class overall SMC of congestion amounts to

$$\mathrm{SMC} = \sum_p \sum_v x_{a,v,p} \sum_u \alpha_{u,p} x_{a,u,p} \frac{\partial t_{a,u,p}}{\partial x_{a,v}} \quad (7)$$

1.5 Limits of the static analysis

The distinction of several time periods makes a step towards a dynamic model of traffic and congestion. In fact, on an arc where congestion does not extend to saturation (i.e. traffic queues), the static, multi-period analysis is sufficient, and there is no need for a dynamic analysis. The main limit of the static analysis pertains to the formation, development and dissipation of traffic queues, within a time period or from one time period to the next. When applied to a period under saturated flow, the static analysis:

- overestimates the flow x_a during the formation and development of the queue, when in fact it is limited by the capacity flow at the arc exit.
- underestimates the flow during the dissipation of the queue, when in fact it remains equal to the capacity flow until the queue disappears.

As will be shown hereafter, the dynamics of queues is a crucial determinant of congestion cost.

2. DYNAMIC ANALYSIS

This section provides a dynamic analysis of congestion cost along an arc. First, some notations are introduced (subsection 2.1). The dynamic analysis of the SMC is addressed in subsection 2.2, considering multiple user classes and time varying capacity. Practical issues conclude subsection 2.2. In particular, it is shown how available data sets, such as vehicle counts from traffic loops, can be used the purpose of *ex post* evaluations. When *ex ante* economic evaluation is of concern, a dynamic flow model has to be used. This is the topic of subsection 2.3. Finally, a numerical example is provided in subsection 2.4.

2.1 Notations

The traffic state and its evolution along an arc a is described using the following variables and notations:

- h denotes an instant within a period H.
- $X^{+}_{a,u}(h) = \int_{h' \in H, h' < h} x^{+}_{a,u}(h')\, dh'$ denotes the cumulated flow of trips of class u up to instant h at the entry point of arc a.
- $X^{-}_{a,u}(h)$ denotes the cumulated flow of trips of class u at the exit point of arc a.
- $t^{+}_{a,u}(h)$ denotes the arc traversal time, i.e. the amount of time needed by a user of class u to reach the exit point of arc a, when entering a at the entry instant h. $X^{-}_{a,u}$, $t^{+}_{a,u}$ and $X^{+}_{a,u}$ are linked by the following relationship: $X^{+}_{a,u}(h) = X^{-}_{a,u}\left(h + t^{+}_{a,u}(h)\right)$.
- it is assumed that congestion on a is due to a bottleneck at the exit point, i.e for any instant h, $\partial X^{-}_{a} / \partial h \leq \kappa_a(h)$, with $X^{-}_{a} = \sum_u X^{-}_{a,u}$, and $\kappa_a(h)$ the capacity flow rate at instant h.

2.2 Economic analysis

Suppose that a queue exists on a over an interval of exit instants $[h_0; h^*]$ and that this queue vanishes at h^*. At every h in $[h_0; h^*]$ the exit flow rate $\frac{\partial X^{-}_{a}}{\partial h}(h)$ is equal to the capacity flow rate $\kappa_a(h)$. At a given instant $\hbar$ taken in $[h_0; h^*]$, let $y = y^{-}_{a,v}(\hbar)$ be a small perturbation of the output flow due to class v, such that y exits the queue at $\hbar$. For all h in $[\hbar; h^*]$, the element $dX^{-}_{a}(h)$ in the exit flow, initially at position $X = X^{-}_{a}(h)$ in the queue, is shifted to position $X^{-}_{a}(h) + y$ in the queue. If it would have exited the queue at instant $h = h_X$, now $dX^{-}_{a}(h)$ exits the queue at instant $h + \delta h$, where $\delta h = y / \kappa_a(h)$ is the amount of time needed to flow y out of the queue at instant h. The time loss by the class u users in the flow element $dX^{-}_{a}(h)$ at instant h is $dX^{-}_{a,u}(h)\delta h$. By summing over all users in $[X^{-}_{a,u}(\hbar), X^{-}_{a,u}(h^*)]$, the cost induced by y at $\hbar$ on the class u users is:

$$\begin{aligned} C_{a,v,u}(y,\hbar) &= \int_{X\in[X^-_{a,u}(\hbar),X^-_{a,u}(\hbar^*)]} \alpha_u \frac{y}{\kappa_a(h_X)} dX \\ &= \alpha_u y \int_{\hbar<h<\hbar^*} \frac{dX^-_{a,u}(h)}{\kappa_a(h)} \end{aligned} \quad (8)$$

Thus the social marginal cost induced by a marginal user of class *v* user to all users of class *u* amounts to, by exit instant $\hbar$

$$\begin{aligned} \gamma^-_{a,v,u}(\hbar) &\equiv \frac{\partial C_{a,v,u}(y,\hbar)}{\partial y} \\ &= \alpha_u \int_{\hbar<h<H^*_a(\hbar)} \frac{dX^-_{a,u}(h)}{\kappa_a(h)} \end{aligned} . \quad (9)$$

Where $H^*_a(\hbar)$ is defined as the first instant when a queue vanishes after $\hbar$ if such a queue exists, and $H^*_a(\hbar)=\hbar$ otherwise. Eq.(9) can be simplified in two cases of practical interest:

- for a single user class, since $\frac{dX^-_{a,u}(h)}{dh}=\kappa_a(h)$ for all *h* such that a queue exists, Eq.(9) becomes

$$\gamma^-_{a,u}(\hbar) = \alpha_u (H^*_a(\hbar)-\hbar) \quad (10)$$

- in a multi-class context, if the capacity flow rate is constant

$$\gamma^-_{a,v,u}(\hbar) = \alpha_u \frac{X^-_{a,u}(H^*_a(\hbar)) - X^-_{a,u}(\hbar)}{\kappa_a}$$

Formula (10) reads as "the marginal congestion cost of a user leaving a bottleneck at instant $\hbar$ is equal to the queue duration from that instant, times the user average value of time". It is especially important since it states in a very simple way the marginal congestion cost in a dynamic setting, that of a vertical queue bottleneck. It had been stated by Fargier (1983) in the fixed capacity case; our extension to the varying capacity case seems to be original.

The interclass congestion cost inflicted by class *v* to class *u* is evaluated by integrating the marginal SMC on the trips of class *v*:

$$\Gamma_{a,v,u}(H) = \int_{h\in H} \gamma^-_{a,v,u}(h)\, dX^-_{a,v}(h) \quad (11)$$

The overall social congestion cost over *H* is, by aggregation over classes:

$$\Gamma_a(H) = \sum_v \sum_u \Gamma_{a,v,u}(H) \quad (12)$$

It should be noticed that the above formulae can be applied for both *ex post* and *ex ante* evaluations. An *ex post* evaluation requires data coming from field measurements, such as vehicle counts and speed or density records. Indeed, if such data are available at the arc exit, then inputs to Eq.(9) can be estimated rather easily. The speed or density records can be used to estimate the queue start and queue end instants. Timestamps in vehicle counts records can be used to estimate the cumulated exit flow volumes. An *ex ante* evaluation requires the inputs to be provided by a dynamic model, either by aggregation of the outputs of a traffic simulator, or using a simple analytical queue model, as in the coming subsection.

2.3 Vertical queue model

As in *(11)*, let us use a vertical queue model to simulate traffic flow along an arc a with a bounded capacity at its exit. The model has three inputs: a vector $X^{+}_{a,u}(h)$ of cumulated flows entering the arc; the capacity flow rate $\kappa_a(h)$; a vector $t_{0,a,u}(h)$ of minimum traversal time functions. Outputs include: the vector of exit cumulated flows $X^{-}_{a,u}(h)$; a multiclass vector of traversal time functions $t^{+}_{a,u}(h)$; and eventually the queued volume $Q_a(h)$. There are two constraints on the exit flows. The *capacity constraint* imposes

$$\frac{\partial X^{-}_{a}}{\partial h}(h) \leq \kappa_a(h) \tag{13}$$

with $X^{-}_{a} = \sum_u \varepsilon_u X^{-}_{a,u}$ and ε_u the passenger car equivalent of a class u user. The *minimum traversal time constraint* imposes that, for any couple of instants (h_1, h_2),

$$X^{-}_{a,u}(h_2) = X^{+}_{a,u}(h_1) \;\Rightarrow\; h_2 - h_1 \geq t_{0,a,u}(h_1)\,. \tag{14}$$

A vector of traversal time functions $t_{a,u}$ is *acceptable* if the associated vector of cumulated output flow $\chi^{-}_{a,u}$, defined by $\chi^{-}_{a,u}(h + t_{a,u}(h)) = X^{+}_{a,u}(h)$, verifies both constraints. $t^{+}_{a,u}$ is defined as the component wise lower bound in the set of acceptable vectors of traversal time functions. The associated vector of cumulated output flows $X^{-}_{a,u}$ is defined by $X^{-}_{a,u}(h + t^{+}_{a,u}(h)) = X^{+}_{a,u}(h)$. For every exit instant $\hbar$, the queue volume verifies

$$Q_a(\hbar) = \sum_u \varepsilon_u X^{+}_{a,u}(h_u) - X^{-}_{a}(\hbar) \tag{15}$$

with h_u such that $h_u + t_{0,a,u}(h_u) = \hbar$

2.4 Numerical example

Let consider two traffic classes: *lv* for long vehicles and *pc* for passenger cars. The arc *a* has a capacity flow rate κ_a which is constant and taken equal to 3,600 pcu/h. The free flow travel time of class *lv*, denoted $t_{0,a,lv}$, is constant and equal to 0.5h. The free flow travel time of class *pc*, denoted $t_{0,a,pc}$, is constant and equal to 0.33h. A long vehicle is equivalent to 3 p.c.u. The following input flows are taken:

- for long vehicles, a constant flow of 400 veh/h from 4:00 to 10:00.
- for passenger cars, the flow is 1,000 veh/h on [4:00, 6:00], 3,000 veh/h on [6:00, 7:00], 2,600 veh/h on [7:00, 8:00], then 1,500 veh/h on [8:00, 10:00].

The main results are illustrated in Fig.1. Cumulated flows for classes pc and lv are plotted in Fig.1a. The plot's key follows the following convention: cumulated input flows are indicated with a +, cumulated output flows are indicated with a −. The total cumulated output flow is the curve keyed X-. Congestion starts at instant 6.33, when the first vehicles in the flow of passenger cars reach the arc exit. From this instant, the slope of the cumulated output flow remains equal to the capacity flow rate until the queue vanishes, at instant 9.28. The plot Fig.1b shows the evolution of the output flow rates for both classes. It illustrates how the capacity is shared between the flow of passenger cars and the flow of long vehicles during the congestion period. Traversal time functions are plotted in Fig.1c. Last, Fig.1d illustrates the distribution of the interclass congestion cost, as functions of the exit instant.

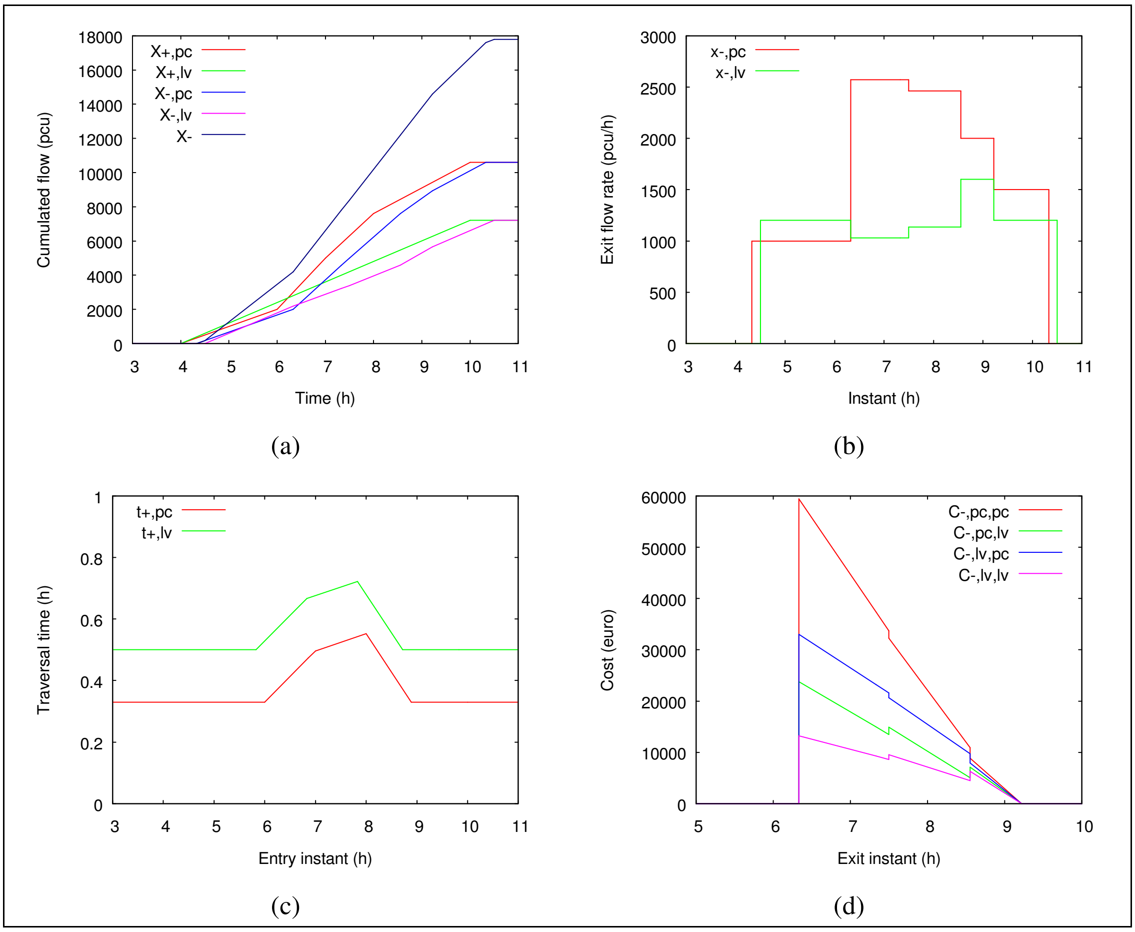


Figure 1: Results from the dynamic analysis: (a) cumulated flows; (b) exit flow rates; (c) traversal time functions; (d) distribution of the interclass congestion costs as functions of the exit instant.

A static model was applied to the same case, as follows:

- BPR time functions with the following parameters; for long vehicles, $t_{0,lv}$ = 0.5 h, γ_{lv} = 1/6, β_{lv} = 1; for passenger cars, $t_{0,pc}$ = 0.33 h, γ_{pc} = 1, β_{pc} = 4.
- capacity flow κ = 3,600 pcu/h and p.c.e. of 3 for long vehicles, consistently with the dynamic model.

In both cases, the congestion costs were evaluated using the following time-to-price tradeoffs: 12 €/h for passenger cars and 40 €/h for long vehicles. A comparison of the results for both models is given in Table 1.

The results of this example clearly indicates that using the static approach to evaluate the social cost of (time lost in) congestion leads to significant underestimates, as compared to the dynamic approach presented in this paper. For the particular set of inputs of the example, figures coming from the static approach are merely half of those coming from the dynamic approach.

Table 1: Comparison of interclass congestion costs (k€).

Class v		*Class u*		
		lv	*Pc*	*lv+pc*
lv	dynamic	22.58	49.12	71.70
	static	37.10	20.36	57.46
pc	dynamic	35.34	85.98	121.32
	static	2.35	43.73	46.08
lv+pc	dynamic	57.92	135.10	**193.02**
	static	39.45	64.09	**103.54**

3. CONGESTION COST BY O-D PAIR IN A TRANSPORTATION NETWORK

This section extends the dynamic analysis of congestion costs along an arc, as stated in section 2, to the case of a transportation network. The first extension deals with the social marginal cost of a single trip along a given route of the network (subsection 3.1). Then, the case of a single origin-destination pair is studied (subsection 3.2). Subtleties arise when considering the distribution of the o-d pair flow on the routes from origin to destination. The purpose of the study by o-d pair is to provide the background for the identification of critical o-d pairs (subsection 3.3). Lastly, an example of congestion cost analysis by o-d pair is provided (subsection 3.4). Notations are introduced when necessary.

3.1 Social marginal cost of a route

Let *G=(N,A)* a digraph, where *N* is a finite set of nodes. An arc $a=(i,j)$ in *A* is a pair of nodes in *N*. *i* is the head node of *a*, *j* is the tail node of *a*. To each arc $a=(i,j)$ is associated an arc traversal time function $t_a^+(h)$ that represents the amount of time needed to reach *v* when leaving *i* at instant *h*. $r=(a_1,...,a_k)$ denotes a route, i.e. a non empty finite sequence of distinct arcs such that the tail node of a_{k-1} is the head node of a_k. If $r=(a,r')$ is a route containing strictly more than one arc, the route traversal time function $t_r^+(h)$ is defined by:

$$t_r^+(h) = t_a^+(h) + t_{r'}^+\left(h + t_a^+(h)\right) \quad (16)$$

If a social marginal cost function $\gamma_a^-(h)$ is associated to each arc *a* in *A* (as exposed in subsection 2.2) the social marginal cost of a route *r* for a given departure instant *h* is:

$$\gamma_r^+(h) = \gamma_a^-\left(h + t_a^+(h)\right) + \gamma_{r'}^+\left(h + t_a^+(h)\right). \quad (17)$$

3.2 Social marginal cost of a o-d pair

Let $i = (o,d), o \neq d$ be a distinguished pair of nodes; R_i the (finite) set of routes between *o* and *d*; $w_i^+(r,h)$ a mapping of *route weights* such that:

$$0 \leq w_i^+(r,h) \leq 1 \text{ and } \sum_{r \in R_i} w_i^+(r,h) = 1$$

Let also assume that an arc traversal cost function $c_a^+(h)$ is associated to each arc *a* in *A*. The traversal cost $c_r^+(h)$ of a route $r = (a, r')$ is defined by:

$$c_r^+(h) = c_a^+(h) + c_{r'}^+\left(h + t_a^+(h)\right) \quad (18)$$

If $X_i^+(h)$ is a cumulated flow of users on the o-d pair *i*, and if users are rational, then the cumulated flow $X_i^+(r,h)$ on each route *r* in R_i is $X_i^+(r,h) = w_i^+(r,h) X_i^+(h)$, with:

$$w_i^+(r,h) > 0 \;\Rightarrow\; c_r^+(h) = c_i^+(h)$$

where $c_i^+(h) = \min\{c_r^+(h), r \in R_i\}$. The *route weights* $w_i^+(r,h)$ express the route choice made by rational users: for every departure *h* instant from *o*, rational users are distributed among the routes from *o* to *d* for which the traversal cost is minimal. Note that the route weights are not uniquely defined. Hence, the SMC of the o-d pair *i* can not *a priori* be taken as the weighted sum of the SMC of routes in R_i, and additional discussion is required.

Let *y* be, at instant *h*, a small perturbation in X_i^+ (i.e. a “rational marginal user”) and assume that the partial derivatives $\frac{\partial c_r^+}{\partial y}$ are known. In order to minimize route costs, the route choice made by *y* must be such that $w_{i,y}^+(r,h)\frac{\partial c_r^+}{\partial y}(h) = w_{i,y}^+(r',h)\frac{\partial c_{r'}^+}{\partial y}(h)$, for all couple (r, r') of routes in R_i. In general, there is no particular relation between the route choice of *y* and the route weights $w_i^+(r,h)$, at the noticeable exception of the two following cases:

- If the route choice $w_i^+(r,h)$ is constant around *h*, and if any route *r* from *o* to *d* such that $w_i^+(r,h) > 0$ is also such that $\frac{\partial c_r^+}{\partial y}(h) > 0$, then $w_i^+(r,h)$ is also a route choice for *y*. In this case, the social marginal cost for the o-d pair *i* is:

$$\gamma_i^+(h) = \sum_{r \in R_i} w_i^+(r,h) \cdot \gamma_r^+(h) \quad (19)$$

- If there exists one route from o to d, say r, such that $w_i^+(r,h)>0$ and $\frac{\partial c_r^+}{\partial y}(h)=0$, then a possible route choice for y is: $w_i^+(r,h)=1$ and $w_i^+(r',h)=0, r'\neq r$. If arc traversal costs increase with arc traversal times, then $\frac{\partial t_r^+}{\partial y}(h)=0$. Since all other routes than r are not part of the route choice of y, and for all routes from o to d, the route traversal time remains constant. In this case, the social marginal cost for the o-d pair i is:

$$\gamma_i^+(h)=0$$

3.3 Definition and identification of critical o-d pairs

When the network under analysis contains a large number of o-d pairs, the ability to identify the most "critical" o-d pairs is of great interest. We shall consider an o-d pair as critical if it is sensitive to network congestion in an acute manner. More precisely, there are two determinants to take into account: first, the specific congestion effect of one flow unit; second, the congestion effect on a distance unit. Thus a relevant indicator is $\chi_i(h)=\gamma_i(h)/D_i(h)$, in which $D_i(h)$ denotes a network distance on the o-d i, defined over the least cost routes from origin to destination for a departure instant h. To keep things simple, let us fix D_i to a constant $D_{0,i}$. For instance, $D_{0,i}$ can be proportional to the straight-line distance between origin and destination.

Then the quantity

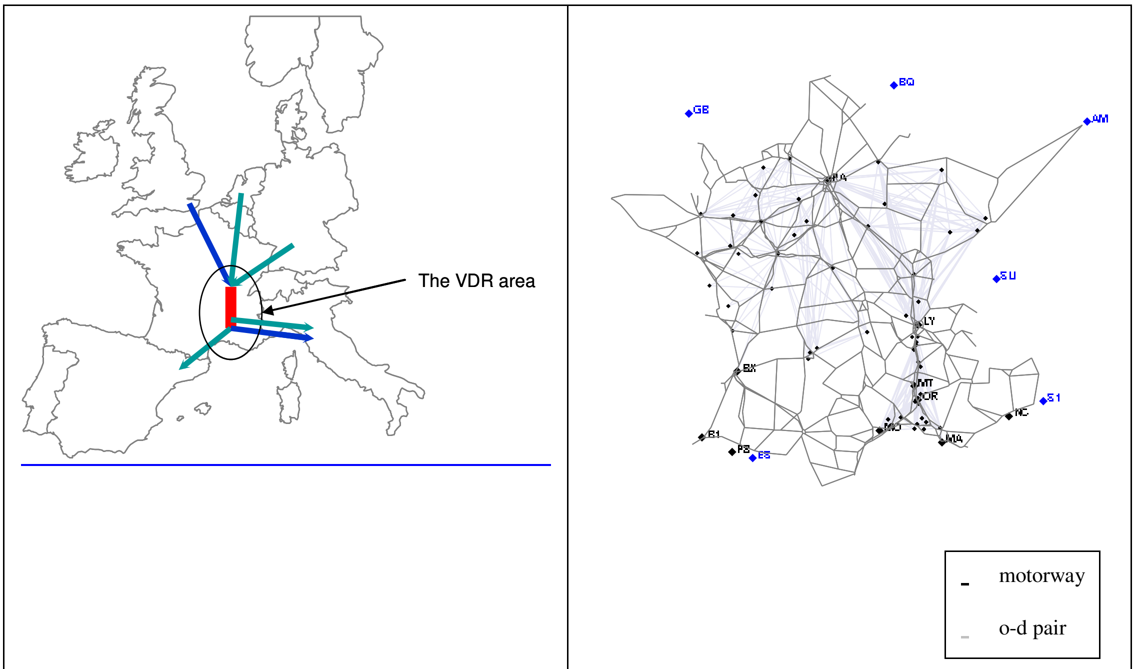
$$\chi_{0,i}(h)=\frac{\gamma_i(h)}{D_{0,i}}$$

interpreted as the SMC per distance unit for the o-d i, allows for a o-d based analysis of the congestion cost on a transportation network. The use of this indicator is illustrated by an example in the next subsection.

3.4 Example

The Vallée du Rhône (VDR) area is of main concern for the French DOT, since a significant part of the trans-european road traffic in Europe concentrates on it. This is particularly true during summer holidays, when tourists coming from northern Europe (including Belgium, the Netherlands, Germany and Great Britain), travel across France to reach (or return from) southern countries (e.g Italy and Spain), meeting on their way people from the Paris area. The situation is depicted in Fig.2. The map on the left hand side (Fig.2(a)) shows the location of the VDR area, together with the structure of traffic flows from foreign countries. The major motorways network is mapped in Fig.2(b), along with the set of o-d pairs this example is concerned with. The main axis in the VDR area in the A7 highway, located between Lyon (**LY** in Fig.2(b)) and Orange (**OR** in Fig.2(b)). The distance between those two cities is around 200km.

Time stamped traffic counts for 628 o-d pairs were provided to us by courtesy of companies of the Vinci Group operating the highway network. They were obtained using toll collection data of July the 14th, 2007. Traffic conditions on the network has been computed using our dynamic traffic assignment model, the Ladta ToolKit *(10)*. The outcome of the assignment is illustrated by Fig.3.



(a) the VDR area (b) Network and trip-end zones.

Figure 2: The major motorway network in France.

Fig.3a to Fig.3d show the simulated traffic conditions on the network for the simulated day, at 0:30 a.m, 6 a.m, noon and 6 p.m. The two main hot spots are the Paris area and the VDR area. Congestion starts before 6 a.m in the VDR area, and the traffic load in this area is particularly heavy around noon. The within the day variations of flow rates computed by the model has been validated with experts of the Vinci Group, by comparison with traffic loops data along some major axis, including the A7.

The simulation of realistic traffic conditions, by using a dynamic traffic assignment model, allows for a fine grain analysis of congestion at the disaggregated level of individual o-d pairs, using the $\chi_{0,i}^{+}$ indicator defined in (3.3). The maps in Fig.4 show the evolution of critical o-d pairs during the day, at 0 a.m , 6 a.m, 12, and 18 p.m. o-d pairs plotted in red are those for which $\chi_{0,i}^{+}$ (i.e. the o-d pair SMC per distance unit) exceeds 1 min/km. The orange colour indicates that $\chi_{0,i}^{+}$ is not null, but less than 1 min/km. These maps clearly indicate *where* and *when* dynamic congestion management measures are more likely to be efficient. Looking at Fig.4a, it appears that, north to Lyon and at the very beginning of the simulated day, five centroids (two of them being very close to each other, near Paris) belong to o-d pairs with a positive SMC/km. This is likely to correspond to traffic flows that will merge later on at some point of the network, and create congestion. And indeed, looking at the congestion map in Fig.3b, congestion exists at 6 a.m north to Lyon. This is quite consistent with the order of magnitude of free flow travel times (e.g. Paris to Lyon is a 5 hours trip).

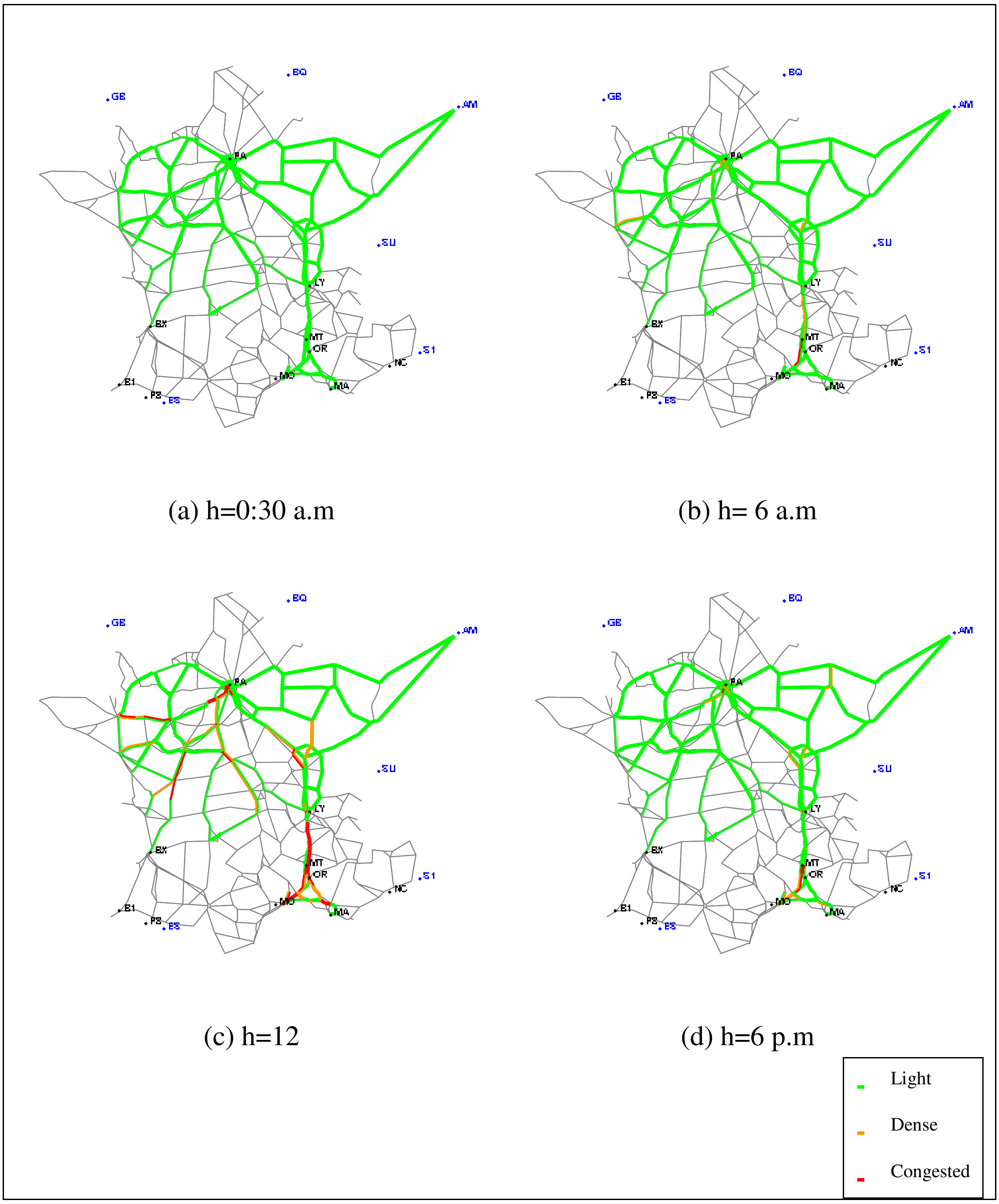


Figure 3: Evolution of simulated traffic conditions during the simulated day.

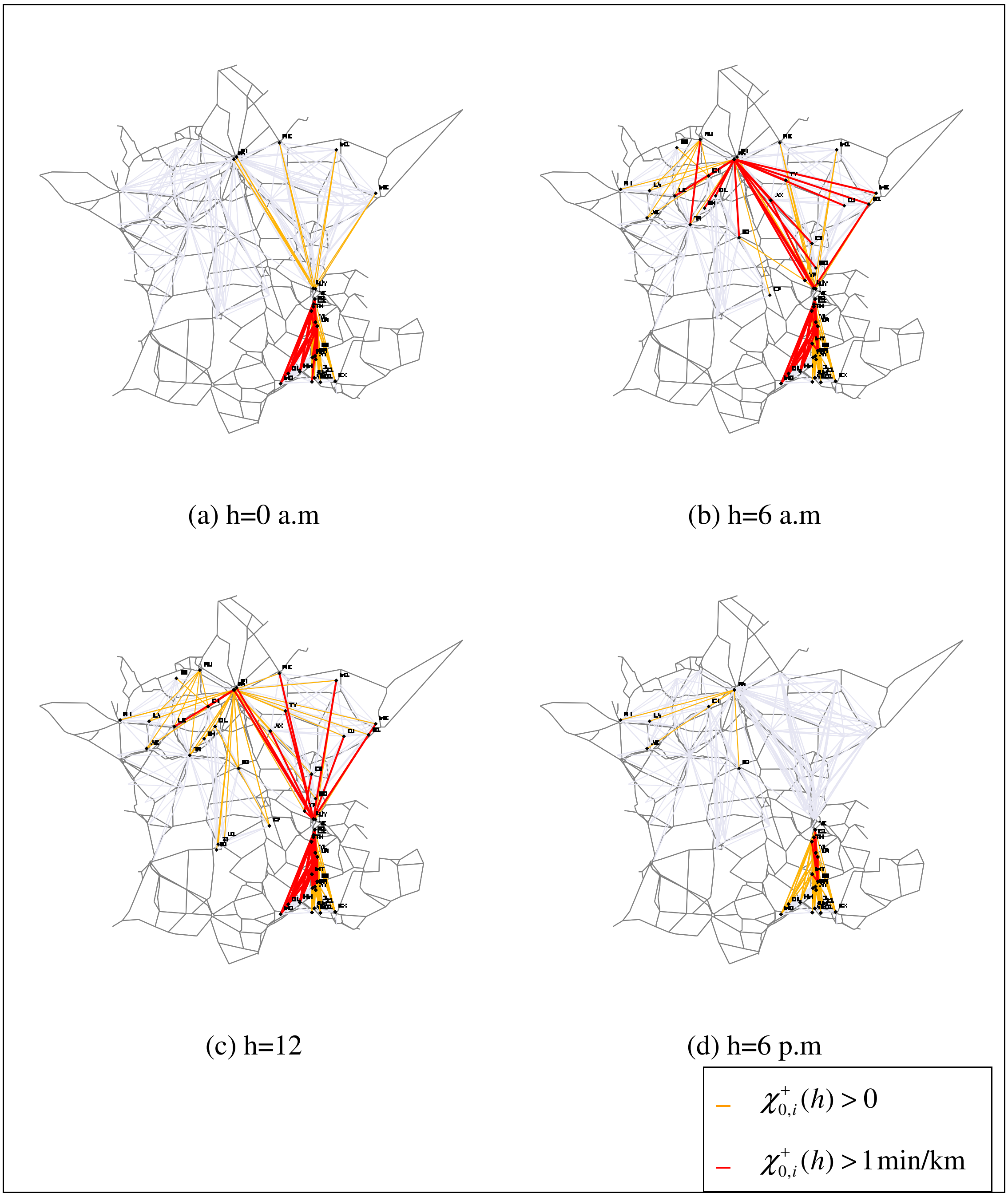


Figure 4: Evolution of critical o-d pairs during the simulated day.

4. SYNTHETIC INDICATORS

Having defined the path and o-d pair SMC, our last issue pertains to the definition of more synthetic indicators of the congestion cost on a wide area. We shall provide zone-based indicators to assess the influence of a zone on the area congestion, as either origin or destination of network trips (subsection 4.1). Then we shall consider sub-network indicators of social congestion cost (subsection 4.2). Lastly, aggregation of the congestion cost with respect to the flows yields indicators of total travel cost that are akin to average cost more than to marginal cost (subsection 4.3).

4.1 Zone-based indicators

As the location of socio-economic activities that induce the trips, a zone may be considered as a determinant of congestion on the transport network. Let us consider a zone o as the origin of the trips destined to zones indexed by d. A synthetic congestion cost from that zone as origin during time interval $H = [h_1, h_2]$ is

$$\gamma_o(H) = \frac{\sum_d \gamma_{od}(H).\Delta X_{od}(H)}{\sum_d \Delta X_{od}(H)},$$

in which $\Delta X_{od}(H) = X_{od}(h_2) - X_{od}(h_1)$ is the volume of demand between h_1 and h_2, and $\gamma_{od}(H) = (\int_{h_1}^{h_2} \gamma_{od}(h)\mathrm{d}h)/(h_2 - h_1)$.

This is measured in units of time or money. To take distance into account, or more precisely to eliminate the dependency upon the distance to travel, a social cost by distance unit is in order:

$$\gamma'_o(H) = \frac{\sum_d \gamma_{od}(H).\Delta Q_{od}(H)}{\sum_d D_{od}(H).\Delta Q_{od}(H)},$$

with $D_{od}(H)$ a network distance (as discussed in subsection 3.3). This is measured in unit of time per distance or money per distance.

Destination-based indicators are easy to derive along the same lines, simply by transposition.

4.2 Sub-network indicators

A sub-network is defined as a subset, A', of the network set of arcs, A.

Congestion cost per flow unit along sub-network A' is defined as the following indicator:

$$\gamma_{A'}(h) = \frac{\sum_{a\in A'} \gamma_a(h).x_a^+(h)}{\sum_{a\in A'} L_a.x_a^+(h)},$$

The weighting by the arc flow, $x_a^+(h)$, is necessary to reflect the distribution of traffic along the sub-network in a statistically representative way. Note that the static counterpart of $\gamma_{A'}(h)$ is

$$\bar{\gamma}_{A'} = \frac{\sum_{a \in A'} x_a^2 \frac{dt_a}{dx_a}}{\sum_{a \in A'} x_a L_a},$$

since $\bar{\gamma}_a = x_a \frac{dt_a}{dx_a}$ in the static model. This differs from the "naïve" formula that follows, which would be erroneous:

$$\tilde{\gamma}_{A'} = \frac{1}{|A'|} \sum_{a \in A'} x_a \frac{dt_a}{dx_a},$$

4.3 Aggregation with respect to flow

It makes little sense to aggregate the SMC with respect to a traffic volume, since a non-negligible volume is likely to yield traffic impacts in a non-linear way. Thus, at the overall level of a network state, a relevant indicator of congestion cost is the total travel time:

$$\gamma_A(H) = \sum_{a \in A} \int_{h1}^{h2} t_a(h) \mathrm{d}X_a^+(h)$$

This is an overall cost, not to be mistaken with a synthetic marginal cost. Indicator γ_A may be used to compare alternative network states in a planning study.

5. CONCLUSION

To sum up, methodological guidance was provided to define and evaluate the social marginal cost of congestion in both the static and dynamic setting and at several spatial levels, ranging from arc to sub-network passing by path, o-d pair and trip-end zone. A simple formula has been provided for the marginal cost in a bottleneck with time-varying capacity. In the dynamic setting, it is crucial to address the propagation of flow in time and space by composition of the arc traversal times along routes. o-d pair based analysis of the SMC was demonstrated in the case study of the VDR area, with courtesy data from the Vinci Group.

Topics for further research include:

- the definition and evaluation of the SMC using more elaborate dynamic traffic model than the vertical queue. A first attempt in this direction, dealing with queue spillback and shock-wave propagation on a single arc, can be found in *(12)*.
- the definition and evaluation of the SMC at junctions crossed by several traffic streams. This area seems to be unexplored so far.

BIBLIOGRAPHY

(1) Vickrey W.S. (1969). Congestion Theory and Transport Investment. *American Economic Review*, Vol. 59, pp. 251-260.

(2) Gazis DC (ed) (1974). Traffic Science. Wiley, New York.

(3) Arnott R., De Palma A. and Lindsey R. (1990). The Economics of a Bottleneck. *Journal of Urban Economics*, Vol. 27, pp. 111-130.

(4) Anderson D. and Mohring H. (1997). Congestion Costs and Congestion Pricing. In Greene DL, Jones DW, and Delucchi MA (eds.), *The Full Costs and Benefits of Transportation: Contributions, Theory and Measurement*. Springer, New York.

(5) B.W. Wie and R.L. Tobin (1998). Dynamic congestion pricing models for general traffic networks, *Transportation Research Part B* 32 (5) (1998), pp. 313–327.

(6) Carey M. and Srinivasan A. (1993). Externalities, Average and Marginal Costs, and Tolls on Congested Networks with Time-Varying Flows. *Operations research*, No. 1, January-February 1993, pp. 217-231

(7) Small KA (1992). *Urban Transportation Economics*. Harwood Academic publishers.

(8) Bureau of Public Roads (1964*). Traffic Assignment Manual*. U.S. Dept. of Commerce, Urban Planning Division, Washington D.C.

(9) Heinz Spiess (1990). Conical Volume-Delay Functions. *Transportation Science*, Vol 24., No. 2.

(10) Aguiléra V. and Leurent F. (2009). On Large Size Problems of Dynamic Network Assignment and Traffic Equilibrium: Computational Principles and Application to Paris Road Network. To appear in *Transportation Research Record*: Journal of the Transportation Research Board.

(11) Leurent F. (2003). On network assignment and supply-demand equilibrium: an analysis framework and a simple dynamic model. Paper presented at the 2003 European Transport Conference (CD Rom edition).

(12) Leurent F. (2005). A dynamic traffic model for the economic analysis of congestion. *Routes/Roads*, No. 325, pp. 46-53.